22.520 Numerical Methods for Partial Differential Equations

Homework # 1 : Due Weds. February 13th

This homework comprises 1 question.

1. In this homework, you will solve a simple 1-Dimensional finite difference problem representing heat transfer in a uniform bar:

$$\nabla^2 T = f(x) = 1, \text{ on the domain } x \,\epsilon\, [0, 1] \tag{1}$$

In this case, the $f(x)$ represents the heat flux per unit length along the bar.

(a) Use one of the methods described in class to derive a one-sided, second order accurate, finite difference expression(s) for the first derivative that could be used at the boundaries $x = 0$ and $x = 1$ of the domain.

(b) Write a pseudo code (a code layout/design) to solve the above PDE.

(c) Implement your own version of a finite difference solver for the above equation using your pseudo code as a code outline. Set the temperature $T(x) = 0$ at $x = 0$ (Dirichlet boundary condition) and the first derivative of the temperature $\frac{dT}{dx} = 0$ at $x = 1$ (Neumann boundary condition).

(d) Derive the analytical solution to the above equation for these boundary conditions.

(e) Using your code from part c, determine the convergence rate of the maximum error and the convergence rate of the $L-2$-norm error of the solution. Plot the error as a function of grid size, Δx (use an appropriate log plot to illustrate convergence).

(f) Modify your finite difference solver by setting the first derivative of the temperature $\frac{dT}{dx} = 0$ at the **both** end points. What happens when you try to solve the problem? Why?